

WIND VELOCITY AND ELEVATION.

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OBSERVATIONS.

Everyone knows that the wind increases with increase of elevation. Even casual observations of such objects as sails of ships, tops of trees, columns of smoke, or isolated clouds suffice to show qualitatively that wind velocity increases with height above the surface: while measurements made by triangulation on freely drifting clouds and balloons, or by anemometers on tethered kites, fully support the conclusions reached by the simpler methods just mentioned. Near the surface of the earth—up to from 2 to 8 meters over an open plane—the condition of the wind, upon whose force this limit depends, may be summarized as follows:

Actual velocity: Exceedingly irregular.

Average velocity: Increases rapidly with elevation.

Rate of velocity increase: {*a*, Increases with average velocity.
 {*b*, Decreases with elevation.

Above the turbulent surface layer the wind increases so nearly regularly with elevation that its approximate velocity at any level up to 16 meters may be computed, according to Stevenson,¹ from its observed velocity at some other height by the empirical equation,

$$V = v \sqrt{\frac{H + 72}{h + 72}}$$

in which V is the computed wind velocity for the level H in terms of the known velocity v at the height h , both elevations being expressed in feet.

If the heights are given in meters this equation becomes

$$V = v \sqrt{\frac{H + 22}{h + 22}}.$$

Other empirical equations expressing the relation of wind velocity to elevation have been given for greater heights. Douglass,² for instance, finds that his velocity observations between 100 meters and 600 meters elevation fairly satisfy the simple equation

$$\frac{V}{v} = \left(\frac{H}{h}\right)^{\frac{1}{4}}.$$

Shaw³ suggests, as a likely formula,

$$V = \frac{H + a}{a} V_0,$$

in which V is the wind velocity at the height H above ground, V_0 the observed anemometer velocity and a a constant obviously depending upon surrounding topography, anemometer exposure and, perhaps, other factors.

Among the most interesting observations on the relation of wind velocity to altitude are those of Dr. Cesare Fabris,⁴ based on some 200 pilot balloon flights made at nearly equal intervals during the year June, 1910—May, 1911, at Vigna di Valle, principal aerological station of the Royal Italian Oceanographic Committee. The co-

ordinates of this station are: Lat. 42° 04' 41" N.; long. 12° 12' 43" E.; altitude, 272.4 meters. It therefore is about 40 kilometers northwest of Rome.

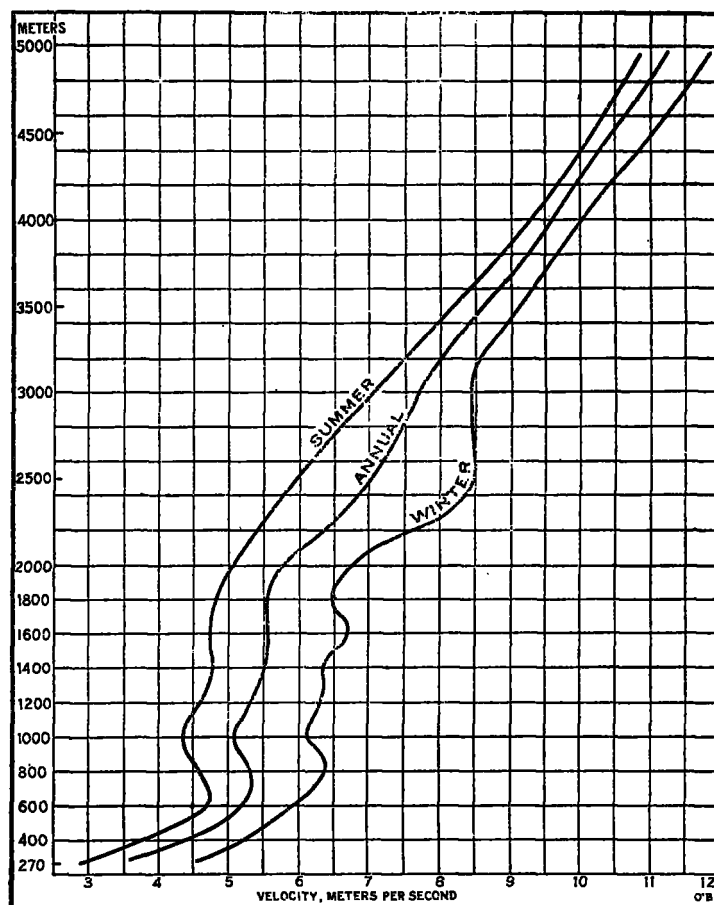


FIG. 1.—Relations of wind velocity to elevation, after Fabris.

The general results of all the observations are summed up in figure 1, which shows four distinct regions:

a. The region of rapid linear increase of velocity with increase of altitude; extending from the surface (272 meters above sea level), where the velocity is least, to an elevation of 600 to 700 meters. This obviously is the region in which the winds are affected by surface friction and the resulting turbulence. Clearly, too, the average number of eddies and their consequent effect on velocity must rapidly decrease with increase of elevation, substantially as indicated by the given velocity-altitude curves.

b. The region of velocity-decrease with increase of altitude; about 100 meters deep and coming immediately above *a*. This decrease probably is due to the mixing of two wind layers, an upper and a lower, moving in very different directions, and, therefore, merely a local and temporary rather than a universal phenomenon.

c. A region of irregular winds slowly increasing with increase of altitude; extending roughly from about 500 to 1,500 meters above the surface. These conditions are of very general occurrence between the levels given.⁵ The irregularity probably is due to that frequent convectional mixing induced by insolation and, at night, by cloud evaporation.

d. A region of approximately constant increase of velocity with increase of elevation; beginning at about

¹ Jour., Scot. meteor. soc., 1880, 5: 248.

² Nature, 1885, 83: 593.

³ Advisory Committee for Aeronautics, Reports and Memoranda, London, 1909, No. 9, p. 8.

⁴ R. Comitato Talassografico Italiano, Memoria 8, pp. 37-46, 1912.

⁵ Berson, Wissenschaftliche Luftfahrten, 1900, 8: 205.

1,500 meters above the surface and extending to at least the maximum height observed, 5,000 meters. The wind velocities of this region, being out of the reach both of friction and convectional disturbances, are determined by the prevailing horizontal pressure gradients.

Cloud and balloon observations show that increase of wind velocity with increase of altitude above 1,500 to 2,000 meters elevation, holds practically to the top of the troposphere where the velocity in middle latitudes may amount to as much as 90 meters per second (200 miles per hour), or even more.

At higher levels, that is in the stratosphere, the average velocity is decidedly less.

HORIZONTAL PRESSURE GRADIENT AND ELEVATION.

All these facts are well known, but there are no generally accepted and satisfactory discussions of the reasons why the average wind velocity at levels above the limit of appreciable surface influence, should go on increasing with increase of elevation up to the isothermal level and then decrease. Indeed data sufficient for a complete solution of this problem are not yet available, and it is only recently that enough facts have become known to indicate at all clearly the several links in the chain of cause and effect that determines the average atmospheric movements in middle and higher latitudes.

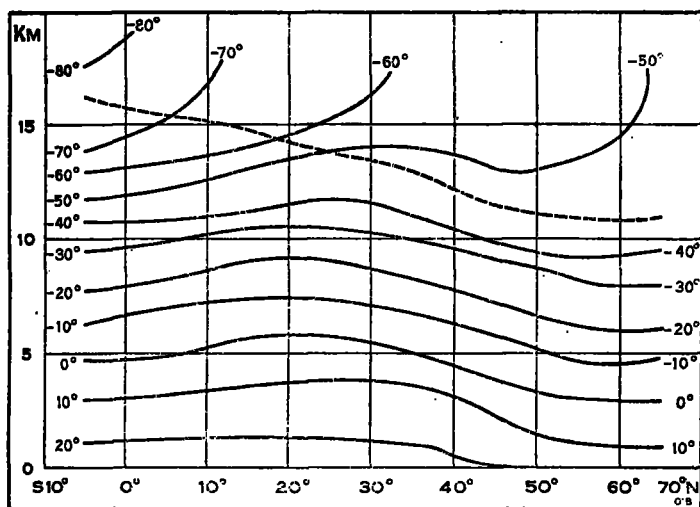


FIG. 2. Relation of temperature to altitude and latitude (N. Hem., Summer), after Süring.

Because of the actual distribution of insolation over the earth the temperature of the lower atmosphere, as shown by observation, is warmest, on the average, in equatorial regions and coldest beyond the polar circles, with intermediate values over middle latitudes. Hence, since the temperature of the air above the earth depends mainly upon convection and radiation from below, it follows that the latitude distribution of temperature in the upper air must be substantially the same as that at the surface, that is, warmest within the tropics and coldest in the polar regions, with intermediate values between. And this, indeed, according to kite and balloon records, does apply at each level up to 10 to 12 kilometers, or to fully three-fourths of the air mass. At much higher levels, 15 to 20 kilometers, for reasons that need not be discussed here, the rare atmosphere is coldest over equatorial regions and warmest over high latitudes. This inverse condition, however, does not apply to the

winter and summer atmospheres of the same place, nor to those of neighboring places on approximately the same latitude.

As a crude first approximation to conditions as they actually exist, assume (1) that the temperature distribution is the same along all meridians, (2) that the temperature change from one latitude to another is the same for all levels, and (3) that sea-level pressure is the same at all latitudes. Assumption (1) approximates the conditions over much the greater portion of the Southern Hemisphere, but, on account of the irregular distribution of land and sea, has to be modified for any detailed study of the winds of the Northern Hemisphere. Assumption (2) conforms roughly to average conditions between the thermal equator and latitude 50°–60°, except near the surface and at altitudes above 10 to 12 kilometers. This is well shown by figure 2, referring to the Northern Hemisphere during its summer, and copied from Süring's paper⁶ on the present state of knowledge concerning the general circulation of the atmosphere. Assumption (3), as applied to normal pressure, is also approximately true except for restricted areas, whose secondary and local effects will not here be discussed.

Consider an atmosphere of the same composition throughout and having initially the same temperature at any given elevation resting on a horizontal plane. Let the temperature be uniformly increased from north to south, say, and by the same amount from top to bottom, thus simulating the temperature distribution that actually obtains in the earth's atmosphere over middle latitudes, as above explained. Find the resulting horizontal pressure gradient at the different levels.

At the height h the horizontal pressure gradient,

$$\frac{dp}{dn},$$

obviously directed from the warmer toward the colder region, is very approximately given by the equation—

$$-\frac{dp}{dn} = p \frac{\Delta h}{HL},$$

in which L is any given horizontal distance along which dn is taken, p the pressure at the level h , above the colder end of L , Δh the difference of vertical expansion of the air below the level in question at the ends of L , or difference of distance through which the level whose original pressure was p was lifted at these two places, and H the virtual height of the atmosphere, approximately 8 kilometers, or height it would have above any point if from there up it had the density which obtains at that point. The negative sign is used because the pressure decreases as n , measured from a warmer toward a colder region, increases. For simplicity let L be in the direction of maximum rate of horizontal temperature change, north-south, in this case.

Under the assumed conditions,

$$\Delta h = \alpha \Delta T h, \text{ approximately,}$$

in which α is the average coefficient of volume expansion of the atmosphere below the level h , and ΔT the difference of temperature change at the ends of L .

⁶ Zeitschrift der Gesell. für Erdkunde, Berlin, 1913, 600.

At any two levels, then, h and h' , the horizontal pressure gradients in the same direction are given approximately by the respective equations,

$$-\frac{dp}{dn} = \frac{pa^{\frac{1}{2}}\Delta T h}{HL},$$

and

$$-\frac{dp'}{dn} = \frac{p'a'^{\frac{1}{2}}\Delta T' h'}{H'L}.$$

But L may be taken the same in both equations, while a , H , and ΔT generally are not greatly different respectively from a' , H' , and $\Delta T'$. In reality,

$$\frac{H}{H'} = \frac{T}{T'},$$

and a' is slightly greater than a when T' is less than T . But in this case it appears from observations that actually $\Delta T'$ is slightly less than ΔT , so that

$$\frac{\frac{dp}{dn}}{\frac{dp'}{dn}} = \frac{ph}{p'h'}, \text{ approximately.}$$

Again, from the 5 to the 10 kilometer level, and even to some distance below the former and above the latter,

$$\frac{p}{p'} = \frac{h'}{h}, \text{ roughly.}$$

Hence, commonly,

$$\frac{\frac{dp}{dn}}{\frac{dp'}{dn}} = \frac{h'h}{hh'} = 1, \text{ approximately.}$$

That is, through these levels or from below 5 kilometers to above 10 kilometers the horizontal pressure gradient established by the temperature difference between adja-

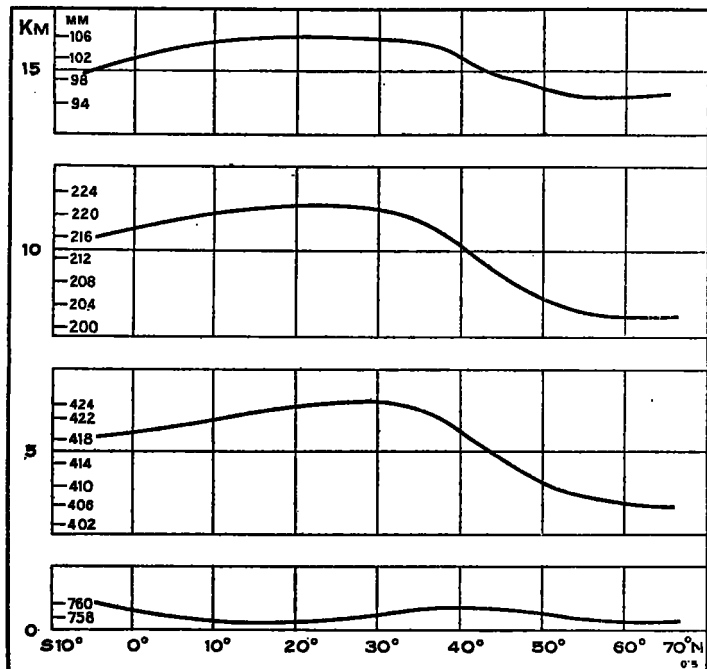


FIG. 3.—Relation of pressure gradient to altitude and latitude (N. Hem., summer), after Süring.

cent regions of air is roughly constant. This conclusion is fully supported by observations as shown by figure 3, referring to the Northern Hemisphere during its summer, and also copied from Süring's paper.⁷

⁷ Süring, op. cit.

LEVEL OF MAXIMUM HORIZONTAL PRESSURE GRADIENT.

The approximate level of the maximum horizontal gradient may be found as follows:

As just explained, in the equation

$$-\frac{dp}{dn} = \frac{pa^{\frac{1}{2}}\Delta T h}{HL},$$

the factor

$$\frac{a^{\frac{1}{2}}\Delta T}{HL}$$

is roughly constant. Writing G for the gradient and K for the "constant," the equation takes the form,

$$G = Kph.$$

Hence G has a maximum value when

$$p dh = -h dp.$$

But,

$$-dp = p \frac{dh}{H}.$$

Hence the pressure gradient is steepest when

$$p dh = \frac{h}{H} p dh,$$

that is, when $h = H = 8$ kilometers, roughly.

The following is a slightly different method of arriving at the same conclusion:

The maximum horizontal pressure gradient resulting from a constant temperature difference between two neighboring columns of air obviously is at that level at which the vertical pressure is most changed by the expansion of the air below due to a constant temperature increase.

Let h be any height, and let a be the average coefficient of volume expansion of the air below this level. Then

$$\Delta h = a^{\frac{1}{2}} h, \text{ nearly,}$$

and

$$\Delta p = \rho g \Delta h = \rho g a^{\frac{1}{2}} h, \text{ nearly,}$$

in which ρ is the density of the air at the level h , and g the local gravity acceleration.

But $\rho = Cp$, in which C is a constant, and

$$\Delta p = p C g a^{\frac{1}{2}} h = K p h,$$

say, in which K may be regarded as a constant.

Hence, as before, Δp has its maximum value when

$$p dh = -h dp = \frac{h}{H} p dh.$$

That is, the horizontal gradient is steepest when $h = H$. But, as is well known, $H = 8$ kilometers, approximately. Hence the horizontal pressure gradient resulting from a temperature distribution substantially that which actually obtains in the atmosphere, is greatest at a height of about 8 kilometers.

CONSTANCY OF MASS FLOW—EGNELL'S LAW.

At a distance above the surface of the earth sufficiently great to avoid appreciable retardation due to friction and turbulence, that is, at elevations greater than 2 kilometers (usually less), the wind obviously must blow in such direction and with such velocity that there is an approxi-

mate equation between the pressure gradient on the one hand and the combined centrifugal force and deflection force due to rotation on the other. Hence, at these levels, if dp/dn is the maximum horizontal pressure gradient,

$$\frac{dp}{dn} = -\rho V(2\omega \sin \phi + \frac{V}{R} \tan \phi), \text{ approximately,}$$

in which ρ is the density of the air at the level under consideration, V the wind velocity, ω the angular velocity of rotation of the earth, ϕ the latitude, and R the radius of the earth.

A little calculation shows that the second term in the parentheses is always small, except in very high latitudes, in comparison with the first. Thus for a west wind moving 22.4 meters per second (50 mis./hr.), at latitude 45° , the first term is about 30 times greater than the second. Hence, under these conditions,

$$\frac{dp}{dn} = -2\rho V\omega \sin \phi, \text{ approximately.}$$

But, as just explained, the horizontal pressure gradient, dp/dn , is roughly constant between 5 and 10 kilometers elevation. Hence at any given latitude, ρV , the mass flow, or mass of air crossing unit normal area per unit time, tends to remain constant with change of altitude from 4 or 5 kilometers above sea level up to the isothermal region. In other words, through this region, ρV , at altitude h is equal to $\rho' V'$ at altitude h' , nearly. This relation between the density and velocity of the atmosphere at different levels is known as Egnell's law,⁸ determined empirically by himself, as also previously by H. H. Clayton,⁹ from cloud observations. Obviously ρV has a maximum value at that level at which the horizontal pressure gradient is a maximum, that is, at about 8 kilometers above sea level.

RELATION OF VELOCITY TO ALTITUDE ABOVE 5 KILOMETERS.

Obviously, if the temperature is constant, as for simplicity we may assume it to be,

$$\frac{\rho}{\rho'} = \frac{p}{p'}.$$

But, as already seen, under this condition of constant temperature, through a considerable range of altitude—that is, from below 5 to above 10 kilometers—

$$\frac{p}{p'} = \frac{h'}{h}, \text{ roughly.}$$

Hence,

$$\frac{\rho}{\rho'} = \frac{h'}{h}, \text{ roughly.}$$

But, as explained above

$$\rho V = \rho' V', \text{ nearly,}$$

therefore,

$$\frac{V}{V'} = \frac{h}{h'}, \text{ approximately,}$$

or the velocity of the wind through the levels in question is roughly proportional to the altitude.

Above the isothermal level over the regions between the thermal equator and latitude 50° or 60° the horizontal temperature gradient decreases, and presently even reverses, with increase of elevation, as shown by figure 2, and therefore the corresponding pressure gradient also

decreases as shown by figure 3. Hence the mass flow, ρV , likewise decreases with elevation above this critical level. Further, the decrease of the horizontal pressure gradient, and, consequently, of ρV , with altitude in the stratosphere appears usually to be more rapid than that of the density alone, from which it follows that the wind velocity generally must have its maximum value at or below the isothermal level.

SOME RESEARCHES IN THE FAR EASTERN SEASONAL CORRELATIONS.

(FIRST NOTE.)

By T. OKADA.

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I.

1. *Introduction.*—It is a well-known fact that for the Far East the great pressure maximum over Siberia and the deep barometric minimum to the south of the Aleutian Islands are the prominent centers of action of the atmosphere in winter, while the great Pacific anticyclone and the continental low area are their summer counterparts. The weather anomaly of the Far East, especially of this country [Japan] is closely and causally related to the occasional change in the positions and intensities of these atmospheric centers.

In 1910 the author published a short paper containing a number of examples of simultaneous correlations in air temperature and rainfall at some places in the Far East. The present note will give a few examples of the remarkable interdependence existing between the pulsations of the Siberian anticyclonic system in winter and the air temperature anomaly of the east coast of Japan in the following summer. The investigation of this correlation may be of some interest for the solution of the fascinating problem of the climatic forecast which aims at predicting the general character of a coming season months in advance.

2. *Method and data.*—As an index of the intensity of the center of action the absolute value of the barometric pressure at a locality situated in the center is not a suitable one. The true measure of the intensity is the barometric gradient. The barometric readings at the Irkutsk Observatory, lying in the heart of the atmospheric center, may be of great service in estimating the intensity of this center. But the great altitude of Irkutsk requires a large correction to be added to reduce its barometric readings to sea level and this greatly reduces the value of those readings as data for the present investigation. In calculating the barometric gradient the most proper material is the pressure data of the meteorological stations lying near sea level. I have therefore used the results of observations taken at the observatories at Zikawei, Nafa, and Izugahara. The geographical coordinates of these observatories are:

Observatory.	Longitude.	Latitude.	Altitude.	Established.
			Meters.	
Zikawei.....	119° 6' E.	31° 12' N.	7.0	1873
Nafa.....	127° 4' E.	26° 13' N.	10.4	1890
Izugahara.....	129° 16' E.	34° 12' N.	9.2	1886

First were computed the differences between the sea-level pressures at Zikawei and Nafa, and those at Zikawei and Izugahara. Since Nafa and Izugahara are equally distant from Zikawei these differences of pressure may be considered as the components of the required pressure gradient. From these components we have computed

⁸ Comptes rendus, 1903, 186:360.

⁹ Clayton in Amer. met'l. Jour., Boston, August 1893, 10:177.